Matter Effects in Three-Flavor Neutrino Mixing and Oscillations

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Outline

1. Neutrinos in vacuum
2. Matter effects on the mixing parameters
3. The dense matter limit
4. The day-night effect
5. Large matter effects on $\nu_\mu \leftrightarrow \nu_\tau$
6. Summary

Neutrinos

- Uncharged leptons with three flavors ν_e, ν_μ, ν_τ
- SM prediction is $m_\nu = 0$
- Neutrino oscillations indicate:
  1. Neutrinos are massive
  2. Neutrinos mix
- Probes of physics at new energy scales

Neutrino Mixing in Vacuum

- Eigenstates of the neutrino mass matrix are not equivalent to the neutrino flavor eigenstates
- Mass and flavor eigenstates are related through the leptonic analogue of the CKM matrix

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$

Neutrino Mixing in Vacuum (2)

- A global analysis gives

Parameter	Best-fit	3σ confidence
Δm^2_{21} [10^{-5} eV^2]	8.1	7.2-9.1
$\|\Delta m^2_{31}\|$ [10^{-3} eV^2]	2.2	1.4-3.3
θ_{12}	33.2°	28.6°-38.1°
θ_{23}	45.0°	$\geq 34.4^\circ$
θ_{13}	0.0°	$\leq 12.5^\circ$

[Maltoni et.al., New J. Phys. 6:122 (2004)]

- The approximate structure of the mixing matrix is then

$$(|U_{\alpha i}|) \simeq \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & \varepsilon \\ \frac{1}{\sqrt{8}} & \sqrt{\frac{3}{8}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{8}} & \sqrt{\frac{3}{8}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Neutrino Oscillations in Vacuum

- The Schrödinger equation

$$\mathrm{i}\frac{\partial \nu}{\partial t} = H\nu,$$

where $H = U\,\mathrm{diag}(m_1^2, m_2^2, m_3^2)U^\dagger/2E$, governs the neutrino flavor evolution

- In a simple two-flavor scenario, the probability of the transition $\nu_\alpha \to \nu_\beta$ is given by

$$P_{\alpha\beta} = \delta_{\alpha\beta} + (1 - 2\delta_{\alpha\beta})\sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4E}L\right),$$

where $\Delta m^2 = m_2^2 - m_1^2$

Neutrino Oscillations in Vacuum (2)

- In a three-flavor scenario, the probability of the transition $\nu_\alpha \to \nu_\beta$ is given by

$$
\begin{aligned}
P_{\alpha\beta} &= \sum_{i,j=1}^{3} \mathrm{Re}(J_{ij}^{\alpha\beta}) - 4 \sum_{1\leq i<j\leq 3} \mathrm{Re}(J_{ij}^{\alpha\beta}) \sin^2\Delta_{ij} \\
&\quad -2 \sum_{1\leq i<j\leq 3} \mathrm{Im}(J_{ij}^{\alpha\beta}) \sin 2\Delta_{ij} \\
\Delta_{ij} &\equiv \frac{\Delta m_{ij}^2}{4E} L \equiv \frac{m_i^2 - m_j^2}{4E} L \\
J_{ij}^{\alpha\beta} &= U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}
\end{aligned}
$$

- How is this affected by the presence of matter?

Neutrino Evolution in Matter

- Due to coherent forward scattering against electrons, there is an extra term in the effective Hamiltonian:

$$H \longrightarrow H + \mathrm{diag}(V, 0, 0),$$

where $V = \sqrt{2} G_F N_e$

- Hamiltonian is no longer diagonalized by the PMNS matrix, the diagonalizing matrix $\tilde{U}$ is an effective mixing matrix in matter

Matter Effects on the Mixing Parameters

Re-diagonalizing the Hamiltonian in the two-flavor case is simple:

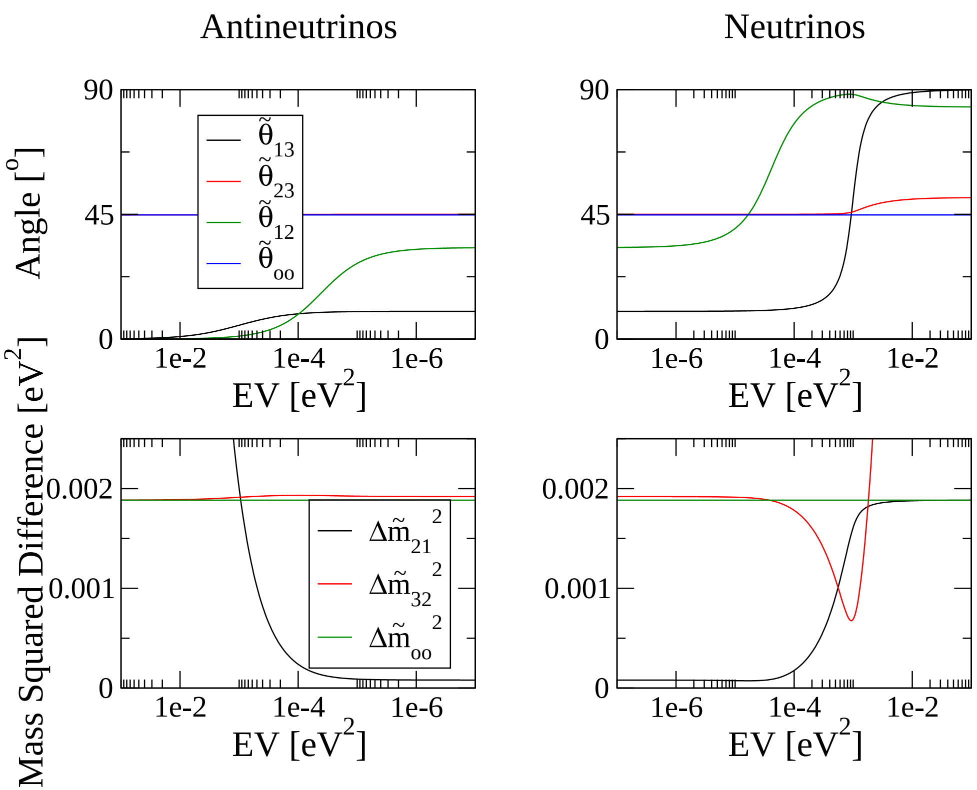
$$\sin 2\tilde{\theta} = \frac{\sin 2\theta}{\sqrt{\left(\cos 2\theta - \frac{2VE}{\Delta m^2}\right)^2 + \sin^2 2\theta}}$$

$$\Delta\tilde{m}^2 = \Delta m^2 \sqrt{\left(\cos 2\theta - \frac{2VE}{\Delta m^2}\right)^2 + \sin^2 2\theta}$$

- Note the so-called MSW resonance at $2VE = \Delta m^2 \cos 2\theta$
- In the three-flavor case, no such easy analytic expression exists for mapping the vacuum mixing parameters to the matter mixing parameters.

Matter Effects on the Mixing Parameters (2)

By numerical simulation:



[Blennow, Ohlsson, hep-ph/0409061]

The Dense Matter Regime

- When $|VE| \gg \Delta m_{ij}^2$, the vacuum Hamiltonian may be regarded as a perturbation
- The ν_e decouples, leaving a two-flavor system
- The result of zeroth order perturbation theory is

$$\sin^2(2\theta_\infty) =$$

$$4\frac{\{s_{23}c_{23}[c_{13}^2 - \alpha(c_{12}^2 - s_{12}^2 s_{13}^2)] - \alpha s_{12}c_{12}s_{13}c_\delta \cos(2\theta_{23})\}^2 + \alpha^2 s_\delta^2 s_{12}^2 c_{12}^2 s_{13}^2}{[c_{13}^2 - \alpha(c_{12}^2 - s_{13}^2 s_{12}^2)]^2 + 4\alpha c_{12}^2 s_{12}^2 s_{13}^2}$$

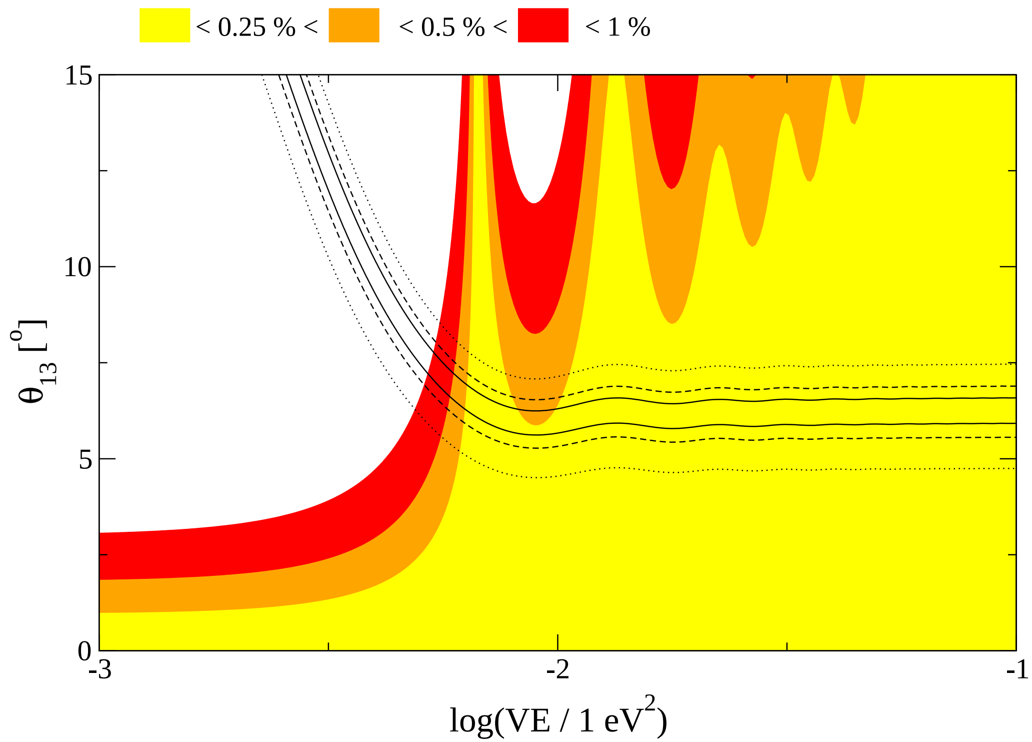
$$\Delta m_\infty^2 = |\Delta m_{31}^2| \sqrt{[c_{13}^2 - \alpha(c_{12}^2 - s_{13}^2 s_{12}^2)]^2 + 4\alpha c_{12}^2 s_{12}^2 s_{13}^2}$$

[Blennow, Ohlsson, hep-ph/0409061]

$$\alpha \equiv \Delta m_{21}^2 / \Delta m_{31}^2$$

The Dense Matter Regime (2)

- The relative accuracy of the dense matter approximation for the ν_μ oscillation probability ($L/E =$ 7000 km / 50 GeV)



[Blennow, Ohlsson, hep-ph/0409061]

The Day-Night Effect

- Thermonuclear reactions in the Sun produce ν_e
- Neutrinos from the sun arrive at the Earth as incoherent superpositions of the mass eigenstates
- The ν_e survival probability is influenced by the Earth matter, giving rise to different survival probabilities during night and day
- This has, until recently, only been studied in a two-flavor framework
- What are the three-flavor effects?

The Day-Night Effect (2)

- The third mass eigenstate effectively decouples, leaving a two-flavor case with effective parameters depending on θ_{13}
- The result is that the probability difference scales essentially as

$$P_N - P_D \propto c_{13}^6,$$

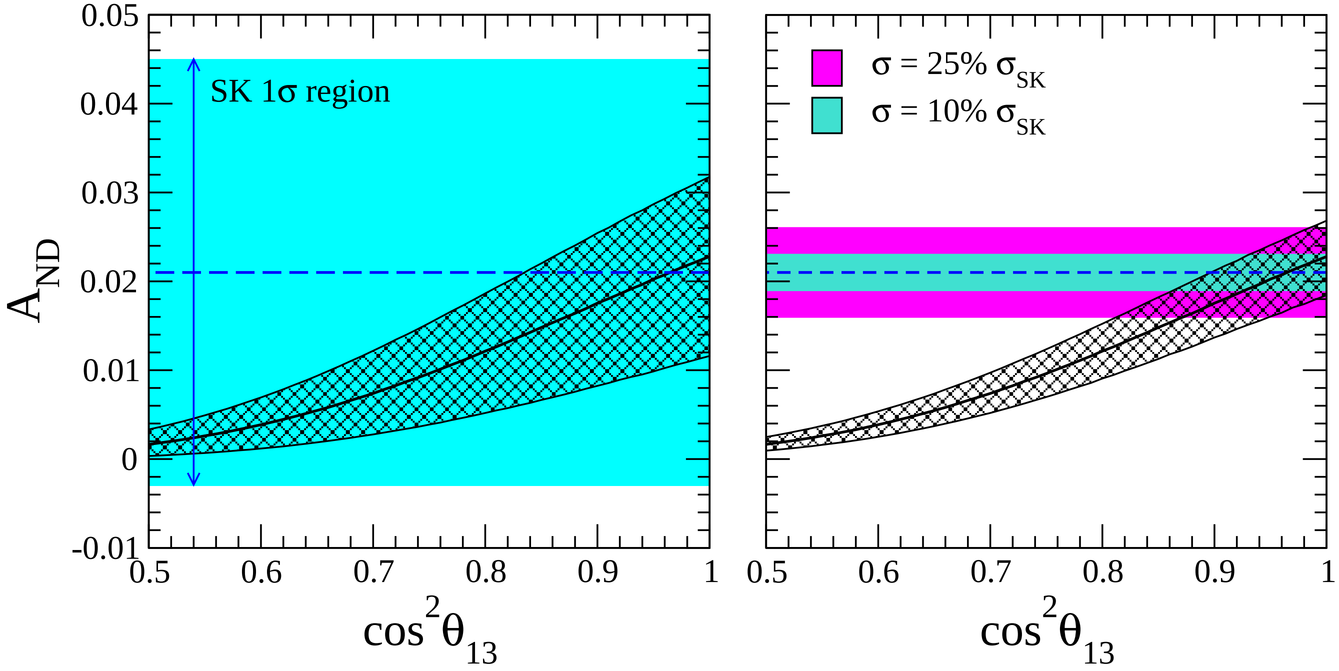
and the day-night asymmetry as

$$A_{n-d} = 2\frac{N-D}{N+D} \propto c_{13}^2$$

[Blennow, et.al., Phys. Rev. D**69**, 073006 (2004)]
[Akhmedov, et.al., JHEP 0405:057 (2004)]

The Day-Night Effect (3)

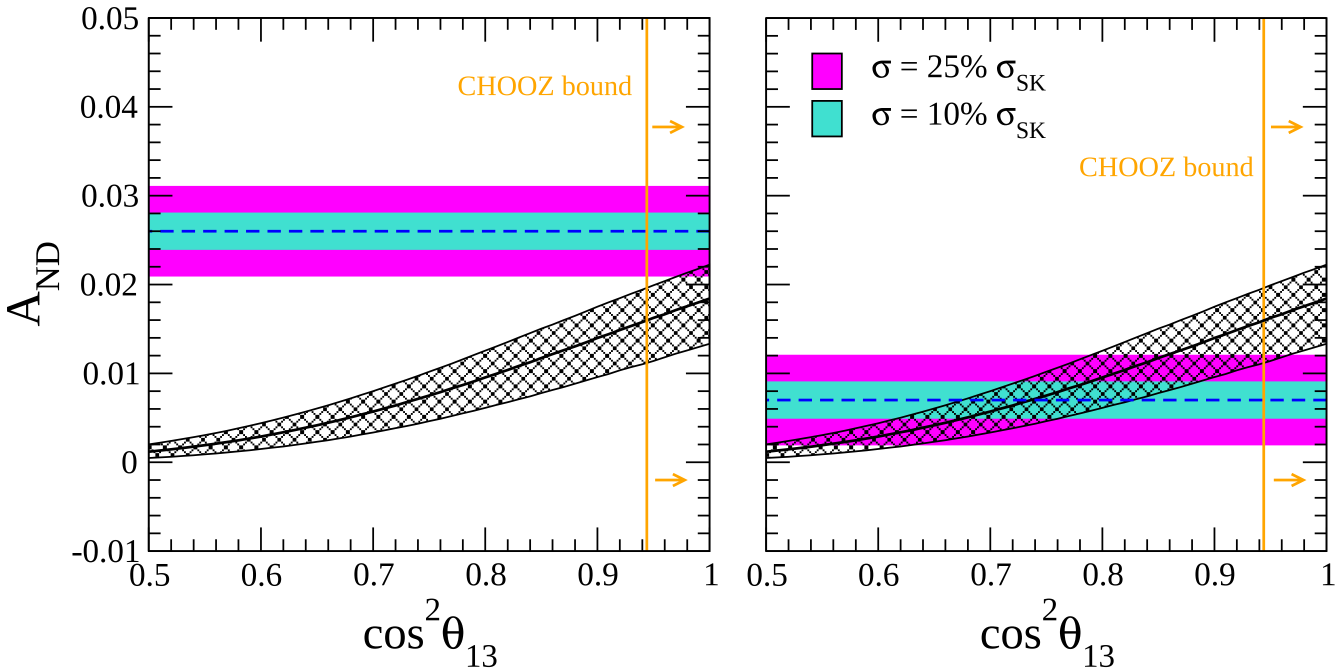
- Could one extract information on θ_{13}?



[Akhmedov, et.al., JHEP 0405:057 (2004)]

The Day-Night Effect (4)

- But the central value might change



[Akhmedov, `hep-ph/0412029`]

Matter Effects in $\nu_\mu \leftrightarrow \nu_\tau$

- In a two-flavor $\nu_\mu \leftrightarrow \nu_\tau$ scenario, there are no matter effects
- This is no longer true in the three-flavor framework unless $\alpha = 0$ **and** $\theta_{13} = 0$
- In particular, at the resonance corresponding to Δm^2_{31}, $\tilde{\theta}_{13}$ is greatly enhanced [Gandhi, et.al, hep-ph/0408361]

Summary

- From neutrino oscillations: Neutrinos are massive and mix
- Matter can affect the behaviour of the neutrino oscillations
- It is important to use three-flavors to analyze neutrino data to probe the full mixing matrix
- We have seen a few examples of this

References

- Blennow, Ohlsson, Snellman, Day-Night Effect in Solar Neutrino Oscillations with Three Flavors, Phys. Rev. D 69, 073006 (2004)
- Blennow, Ohlsson, Effective Neutrino Mixing and Oscillations in Dense Matter, hep-ph/0409061
- Akhmedov, Tortola, Valle, A Simple Analytic Three-Flavor Description of the Day-Night Effect in the Solar Neutrino Flux, JHEP 0405:057 (2004)
- Gandhi, Ghoshal, Goswami, Mehta, Sankar, Large Matter Effects in $\nu_\mu \to \nu_\tau$ Oscillations, hep-ph/0408361