The Solar Neutrino Day-Night Effect

Master of Science Thesis

Mattias Blennow
Division of Mathematical Physics
Department of Physics
KTH

Why This Interest in Neutrinos?

- Massless in SM of particle physics
- Strong evidence for neutrino oscillations
- Probe of physics beyond the SM
- Many open questions

Neutrino Oscillations

- Suppose mixing of neutrinos

$$|\nu\rangle_f = U\,|\nu\rangle_m$$

- Unitary mixing matrix
- Relates flavor and mass bases
- Hamiltonian diagonal in mass basis

$$H_m = p\mathbf{1} + \frac{1}{2p}\begin{pmatrix} m_1^2 & 0 & \dots & 0 \\ 0 & m_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & m_n^2 \end{pmatrix}$$

Two Flavor Oscillations

- Standard parametrization of mixing matrix

$$U = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$$

$$c = \cos\theta \qquad s = \sin\theta$$

- Simple expression for oscillation probabilities

$$P_{ex}(t) = P_{xe}(t) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4E}t\right)$$

$$\Delta m^2 = m_2^2 - m_1^2$$

Three Flavor Oscillations

- Standard parametrization of mixing matrix

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ij} = \cos\theta_{ij} \qquad s_{ij} = \sin\theta_{ij}$$

- No simple expression for oscillation probabilities
- Approximate two flavor oscillations

Matter Effects

- Additional effective term in Hamiltonian

$$H_{CC,\text{eff},f} = \begin{pmatrix} V_{CC} & 0 & \dots \\ 0 & 0 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$V_{CC} = \sqrt{2} G_F N_e$$

- Matter eigenstate basis

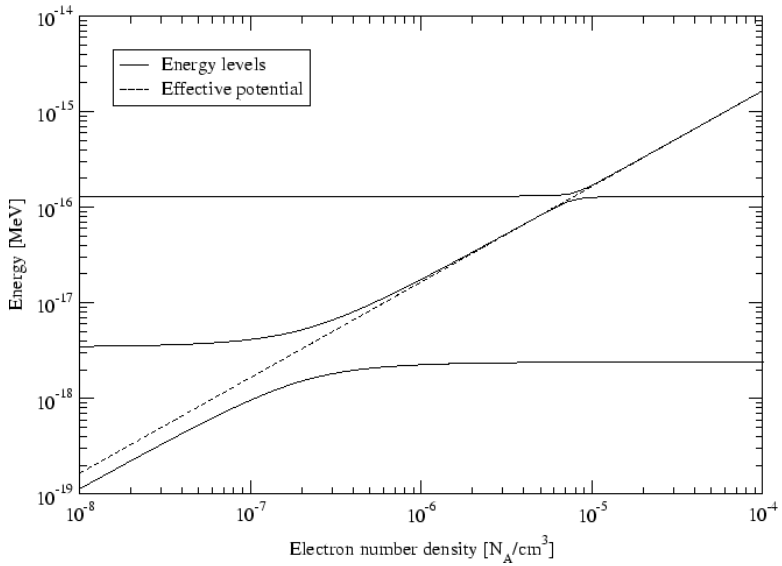
$$|\nu\rangle_f = \hat{U} |\nu\rangle_M$$

Two Flavors in Matter

- Matter basis not fixed in time, induces transitions between matter eigenstates
- Adiabatic approximation
- Resonance
- Mikheyev-Smirnov-Wolfenstein (MSW) effect

Three Flavors in Matter

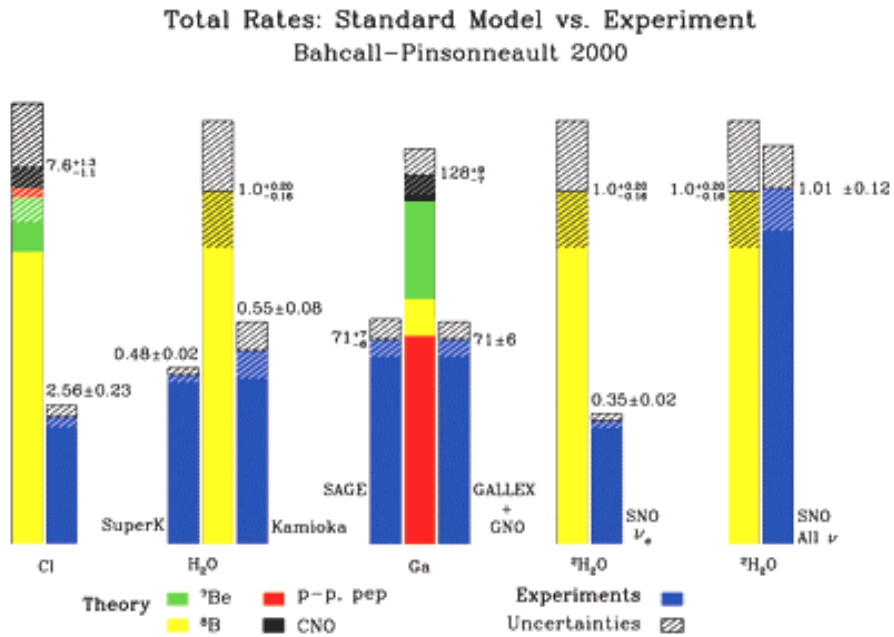
- Two possible resonances depending on sign of mass squared differences
- One resonance if large mass squared difference is negative, second resonance occurs for anti-neutrinos



- Reduces to two flavor resonances if resonances are fairly separated

Solar Neutrinos

- Produced as electron neutrinos by thermonuclear reactions in the Sun according to the Standard Solar Model (SSM)
- Deficit in measured flux at Earth, known as the ”Solar Neutrino Problem”
- Solution: Oscillations of neutrinos to other neutrino flavors



The Day-Night Effect

- Does Earth affect the flux of solar neutrinos?
- The Day-Night asymmetry

$$A_{n-d} = 2\frac{N - D}{N + D}$$

General Approach

- Mass eigenstates arriving at the Earth are incoherent
- Survival probability when arriving at the Earth

$$P_S = \sum_{i=1}^{n} k_i |\langle \nu_e | \nu_i \rangle|^2 = \sum_{i=1}^{n} k_i |U_{ei}|^2$$

- Survival probability after traversing the Earth

$$P_{SE} = \sum_{i=1}^{n} k_i |\langle \nu_e | \tilde{\nu}_i \rangle|^2$$

$$|\tilde{\nu}_i\rangle = |\nu_i(L)\rangle$$

$$P_{ie} = |\langle \nu_e | \tilde{\nu}_i \rangle|^2$$

- No effect if partial fluxes are equal

Solar Production and Propagation With Two Flavors

- General parametrization of partial fluxes

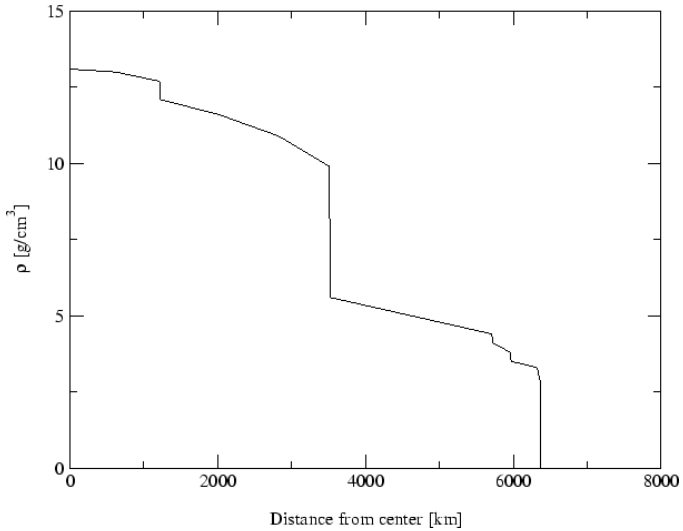
$$k_1 = \frac{1 + D_{2\nu}}{2} \qquad k_2 = \frac{1 - D_{2\nu}}{2}$$

- Averaging over quickly oscillating phases yields

$$D_{2\nu} = \int_0^{R_\odot} \mathrm{d}r\, f(r) \cos(2\hat{\theta}(r))(1 - 2P_{\mathrm{jump}})$$

The Earth Electron Number Density

- Electron number density proportional to matter density profile according to the Preliminary Reference Earth Model (PREM)



- Approximation: Constant electron number density
- Detectors at high latitudes
- Mantle density is fairly constant
- Want to derive analytic expression
- Main non-adiabatic process is entry into the Earth

Two Flavor Propagation in the Earth

- The time evolution matrix is found by exponentiating the Hamiltonian
- The resulting probability is

$$P_{2e} = \sin^2\theta + \frac{KV_E}{4a^2}\sin^2(2\theta)\sin^2(aL)$$

$$a = \frac{1}{2}\sqrt{V_E^2 - 2KV_E\cos(2\theta) + K^2}$$

$$K = \frac{\Delta m^2}{2E}$$

Two Flavor Day-Night Difference

- Difference between day and night survival probabilities

$$P_{n-d} = -D_{2\nu}\frac{KV_E}{4a^2}\sin^2(2\theta)\sin^2(aL)$$

- For the allowed parameter region

$$P_{n-d} \simeq -2D_{2\nu}\frac{EV_E}{\Delta m^2}\sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4E}L\right)$$

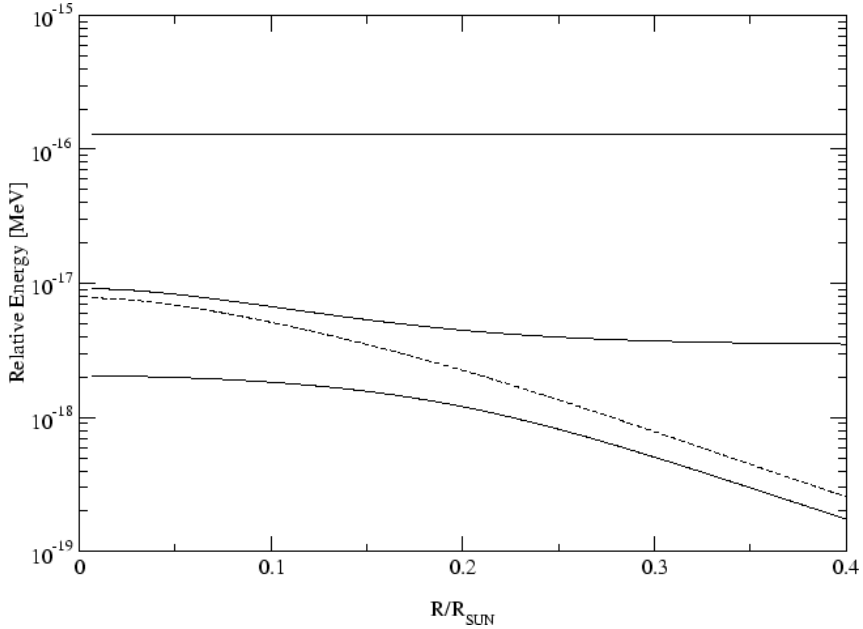
Solar Production and Propagation with Three Flavors

- Only one resonance
- Approximation:

$$k_3 \simeq \sin^2\theta_{13}$$

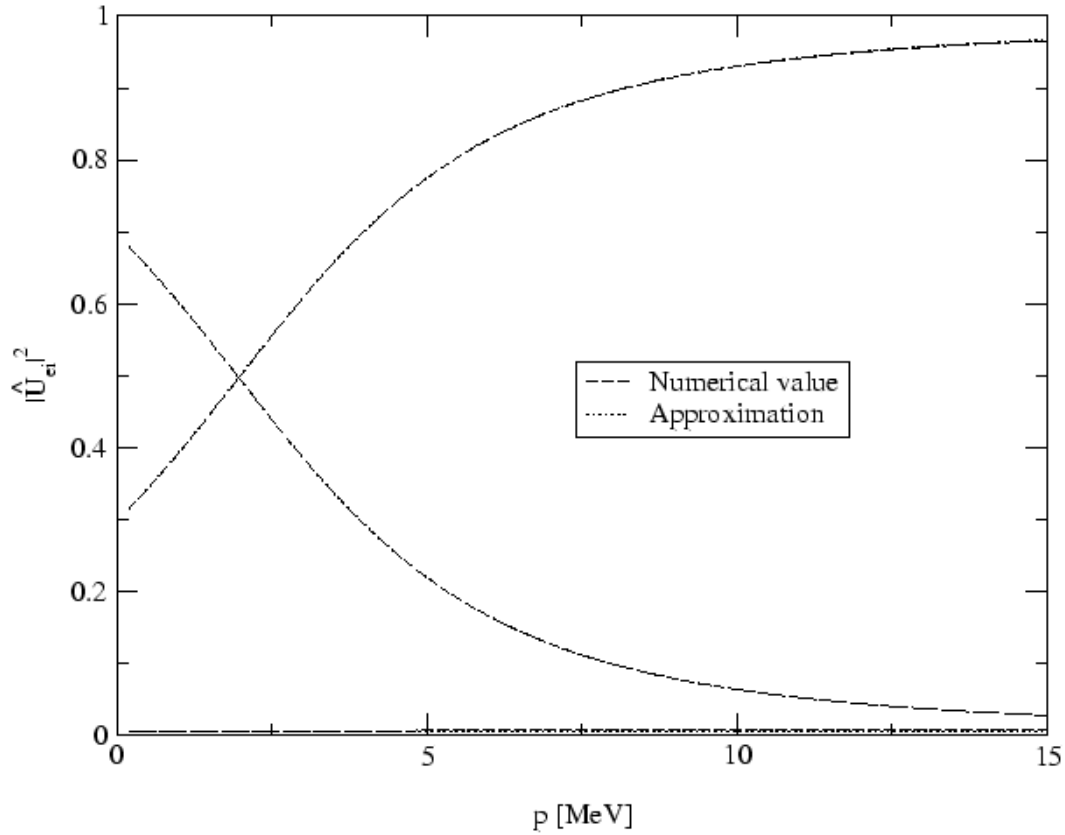
- Approximation: Remaining matter eigenstates evolve as in the two flavor case with effective potential

$$V_{\text{eff}} = c_{13}^2 V_{CC}$$



Solar Production and Propagation with Three Flavors

- This is an **excellent** approximation



Three Flavor Propagation in the Earth

- The Earth matter potential is weak
- Approximation:

$$P_{3e} = |U_{e3}|^2 = s_{13}^2$$

- Remaining matter eigestates evolve as an approximate two flavor case with effective potential

$$V_{\text{eff}} = c_{13}^2 V_{CC}$$

- The resulting probability is

$$P_{2e} = c_{13}^2 s_{12}^2 + c_{13}^4 \frac{K V_E}{4a^2} \sin^2(2\theta_{12}) \sin^2(aL)$$

$$a = \frac{1}{2}\sqrt{c_{13}^4 V_E^2 - 2K c_{13}^2 V_E \cos(2\theta) + K^2}$$

Three Flavor Day-Night Difference

- Difference between day and night survival probabilities

$$P_{n-d} = -c_{13}^6 D_{3\nu} \frac{K V_E}{4a^2} \sin^2(2\theta_{12}) \sin^2(aL)$$

- For the allowed parameter region

$$P_{n-d} \simeq -2c_{13}^6 D_{3\nu} \frac{E V_E}{\Delta m^2} \sin^2(2\theta_{12}) \sin^2\left(\frac{\Delta m^2}{4E} L\right)$$

- We regain the two flavor expression when

$$\theta_{13} \longrightarrow 0$$

The Day-Night Asymmetry at Detectors

- Two types of reactions
- Elastic scattering (ES) - SuperK, SNO

$$\nu_x + e^- \longrightarrow \nu_x + e^-$$

- Charged-current (CC) - SNO

$$\nu_x + d \longrightarrow p + p + e^-$$

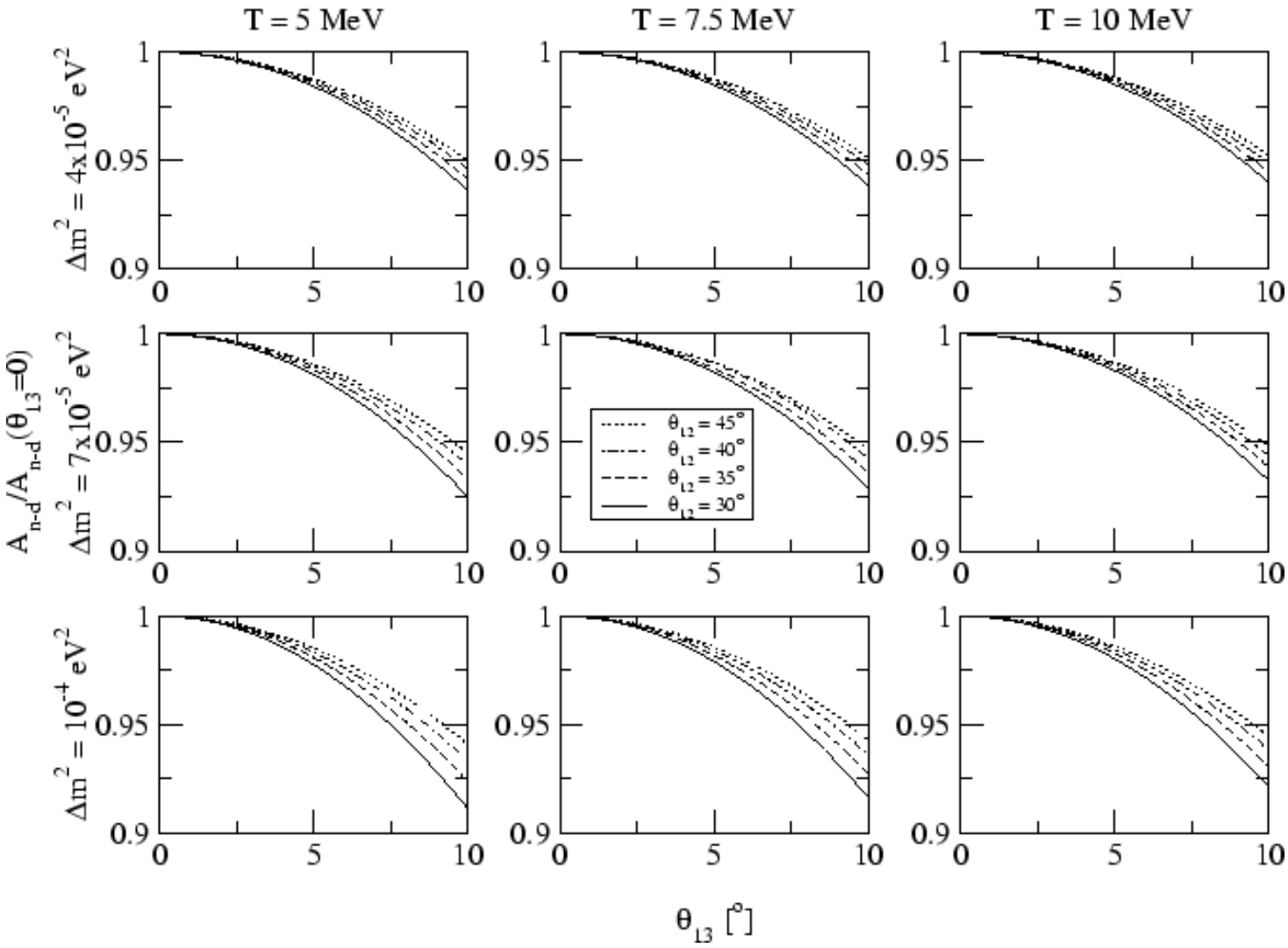
- Day-night asymmetry at specific energy

$$A_{n-d}(T) = 2\frac{N(T) - D(T)}{N(T) + D(T)}$$

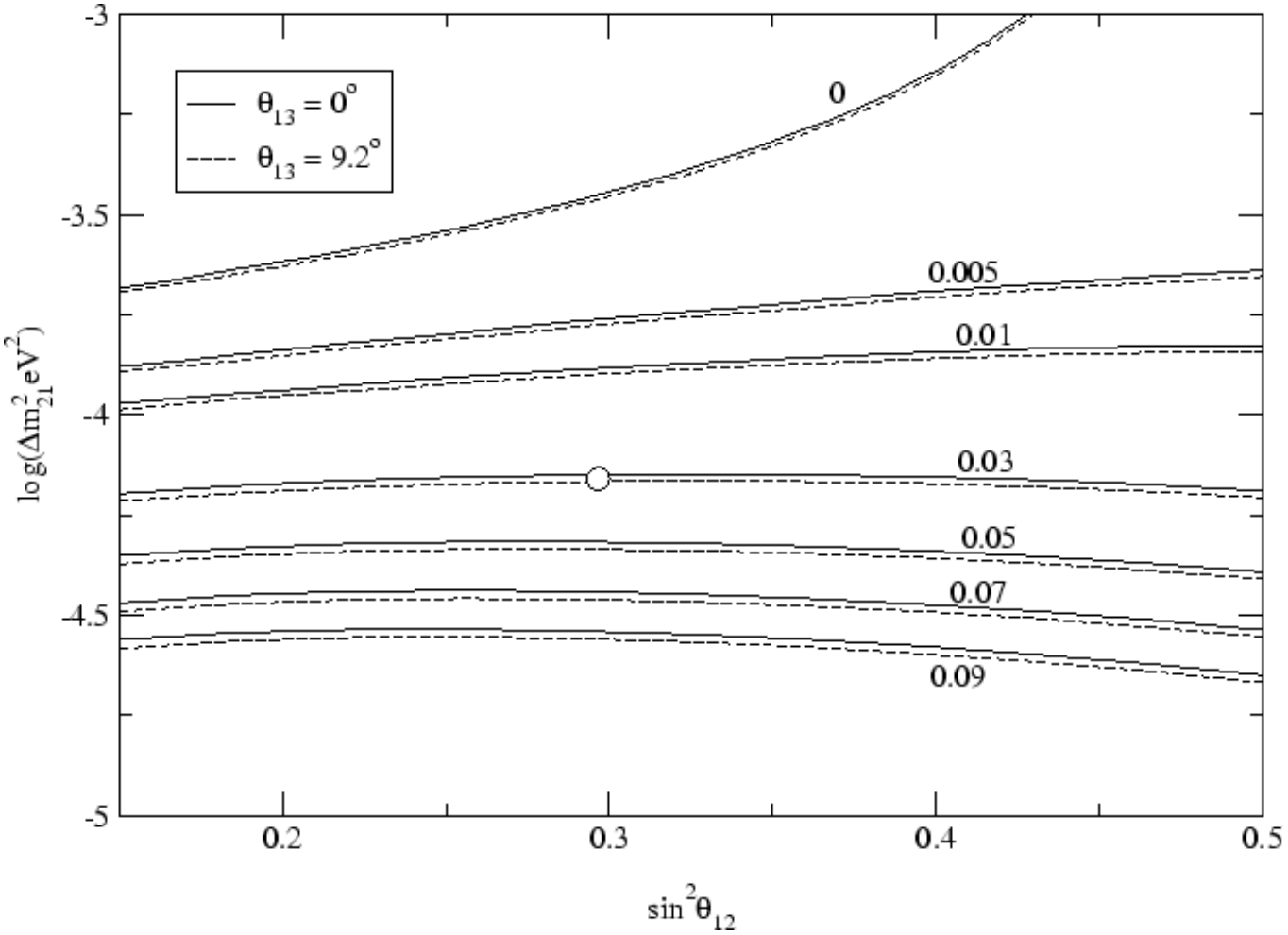
- Or the total day-night asymmetry

$$A_{n-d} = 2\frac{\int_{T_{\text{thld}}}^{\infty} \mathrm{d}T(N(T) - D(T))}{\int_{T_{\text{thld}}}^{\infty} \mathrm{d}T(N(T) + D(T))} = 2\frac{N - D}{N + D}$$

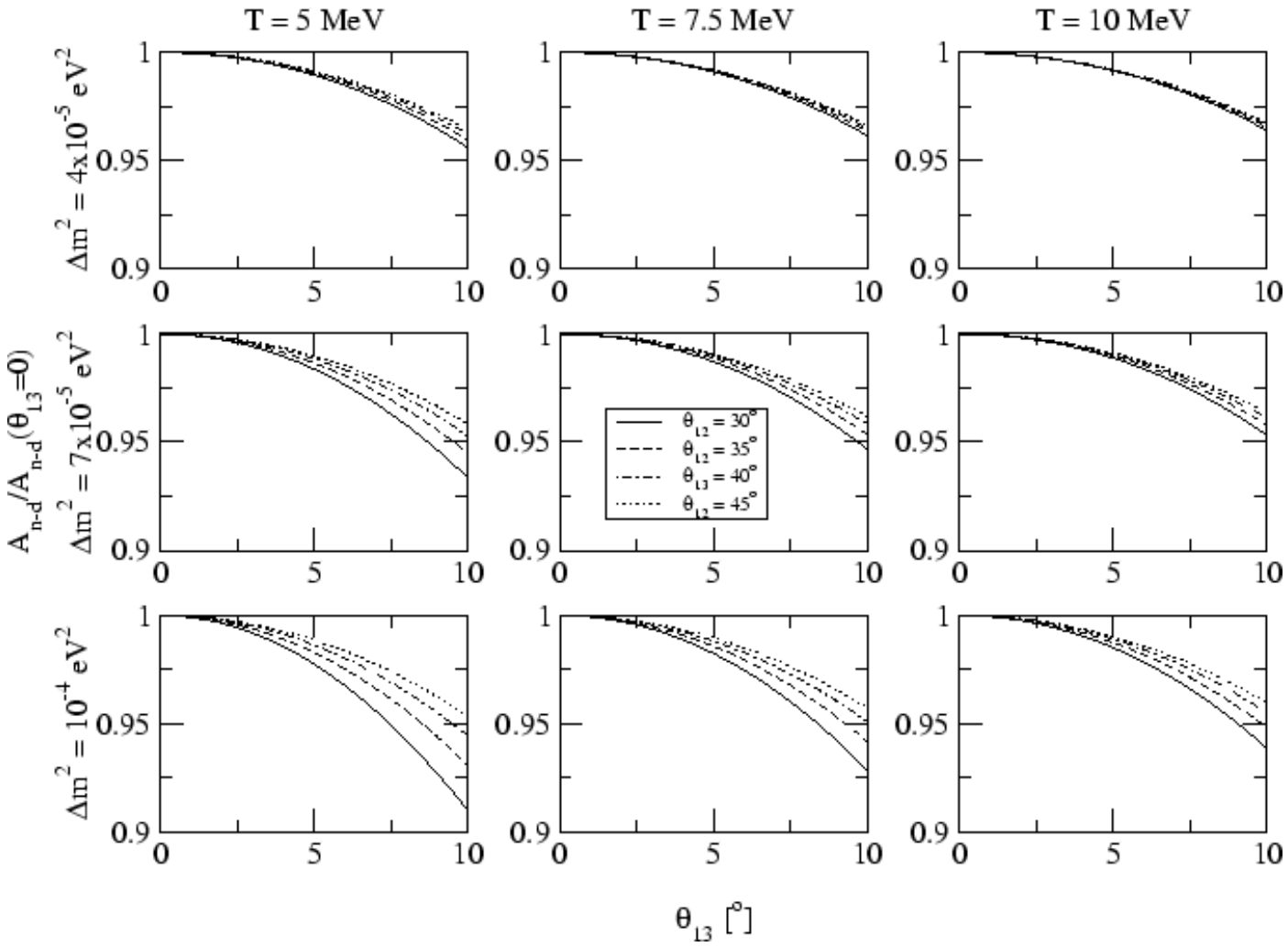
ES results – Specific Energy



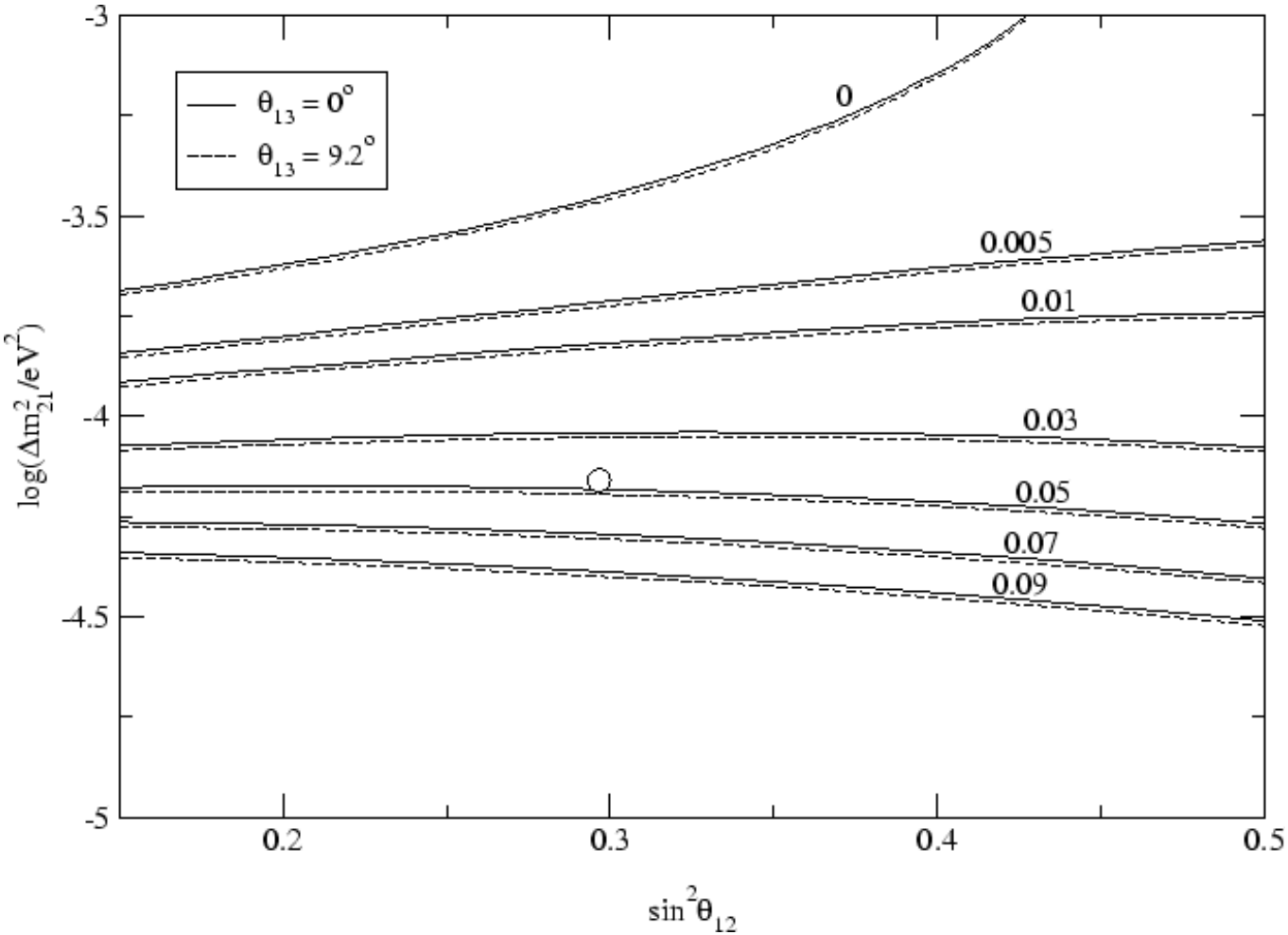
ES results – Total



CC results – Specific Energy



CC results – Total



Neutral-Current Detection

- Is there a day-night asymmetry in the neutral-current reaction?

$$\nu_x + d \longrightarrow \nu_x + p + n$$

- Cross-section equal for all flavors to leading order in weak coupling constant

Summary

- Approximate analytical expressions for day and night solar electron neutrino survival probabilities
- Numerically computed three flavor effects on the day-night asymmetry at detectors
- Effects are small compared to current experimental uncertainties

Thank You For Listening